Exam Subatomic Physics
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Gerco Onderwater

Rijksuniversiteit Groningen / KVI

Before you start, read the following:

- Write your name and student number on top of each page of your exam;
- Illegible writing will be graded as incorrect;
- Final grade = (10 + sum of points)/10;
- *Good luck!*

1 Allowed and Forbidden Processes (15 Points)

Examine the following processes carefully, and state for each one whether it is *possible* or *impossible*, according to the Standard Model. In the former case, state which interaction(s) is(are) responsible – strong, electromagnetic or weak; in the latter case, cite a conservation law that prevents it from occurring. When unambiguous, the charge is not indicated, thus γ, Λ, and n are neutral; p is positive, e is negative, *etc.* (1.5 points for each process)

(a) $e^+ + e^- \to \pi^+ + \pi^- + \pi^0 + \pi^- + \pi^+$ charge OK, via virtual photon

(b) $\bar{\nu}_\mu + p \to \mu^+ + n$ charge OK, anti-nu-mu, weak interaction

(c) $\nu_e + p \to e^+ + \pi^0 + \Lambda$ lepton number violation : L=1 $\to$ L=-1

(d) $e^- + e^- \to \mu^- + \mu^-$ Lepton flavor violation; Lepton number conserving; Weak interaction

(e) $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + e^- + e^-$ Lepton number violation; Majorana neutrinos OK, but not in SM. When interpreted as atoms & charge non-conservation also OK.

(f) $p \to \pi^+ + e^- + e^+ + \gamma$ Baryon number violation (B(p)=1, B(π^+)=0

(g) $^2H \to p + n + \gamma$ Negative energy release. When interpreted as deuterium atom & charge non-conservation also OK.

(h) $\mu^+ + e^- \to \mu^- + e^+$ Lepton flavor violation; Lepton Nr. conservation; Could go via neutrino oscillation

(i) $p + \bar{p} \to b\bar{b}$ OK, strong interaction

(j) $p + {}^8\mathrm{Be} \to {}^8\mathrm{Be} + n + \pi^+ + \pi^- + \pi^+$ OK, strong interaction because hadrons.

2 Cross sections (20 Points)

To study the radioactive decay of ^{20}Na, which has a halflife of about 0.5 s, a sample of ^{20}Na is produced by colliding a beam of ^{20}Ne on a hydrogen target in a two-step fusion-evaporation reaction.

(a) Which nucleus is formed in the fusion (the first step)? Which other particle(s) is (are) produced in the evaporation (the second step)? (2 points) $^{20}\mathrm{Ne}(Z = 10, N = 10) + p(Z = 1, N = 0) \to {}^{21}\mathrm{Na}^*(Z = 11, N = 10)$. Desired particle is ^{20}Na(Z=11,N=9), so the evaporated particle has $(Z = 0, N = 1)$. This is a neutron.

(b) The hydrogen gas target has a length of 10 cm and is kept at liquid nitrogen temperature (T=77.5 K). The gas may be assumed to be an ideal gas (recall that 1 mole of an ideal gas has a volume of 24 liters at 298 K and 1 atm). It is hit by a 1 μA (electrical) fully stripped Neon beam. If the reaction cross section is 500 mb, calculate how many ^{20}Na nuclei are produced per second. (10 points) Production rate = Flux $\times$ Nr.

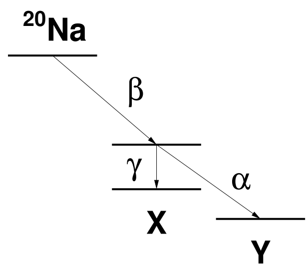
scattering centers illuminated by beam × cross section : $\Gamma = \Phi \cdot N \cdot \sigma$.
Flux = 1 μA / Charge per ion (Z=10) × ions per Coulomb (1C=6.2415096 × $10^{18}e$): $\Phi = 1 \times 10^{-6}/10 \cdot 6.2415096 \times 10^{18} = 6.24 \times 10^{11}/s$.
Scattering centers = Density × length. Ideal gas density : ρ= 1 mole/24l at 298K; density scales as 1/temperature; at 77.5K the density is 298/77.5 times larger; there are 2 nuclei per H_2 molecule :
$\rho = 298/77.5 \times (1\,mole)/(24 \times 10^3\,cm^3) \cdot (6.022 \times 10^{23}/\text{mole}) \cdot 2 = 1.93 \times 10^{20}/cm^3$.
$N = \rho \cdot l = 1.93 \times 10^{20}/cm^3 \cdot 10\,cm = 1.93 \times 10^{21}/cm^2$.
$\Gamma = (6.24 \times 10^{11}/s) \cdot (1.93 \times 10^{21}/cm^2) \cdot 500\,mb \cdot (10^{-27}\,cm^2/mb) = 6 \times 10^8/s$

(c) The incoming Neon beam has a kinetic energy of 25 MeV *per nucleon*. The particles in the final state all have equal velocity. What is the (approximate) kinetic energy of the produced ^{20}Na? You may use that $M(^A_ZY) = A \cdot u$. (3 points) Total energy initial state = Mass(20Neon)+T(20Neon)+Mass(p). Total energy final state = Mass(20Na)+T(20Na)+Mass(n)+T(n). Use Mass(20Ne)=Mass(20Na) and Mass(p)=Mass(n) to find T(20Ne)=T(20Na)+T(n). We have T<<M, so we use classical kinematics : $T = mv^2/2$. Further use Mass(20Na) = 20 Mass(n). So:
$T(Neon) = A \cdot 25\text{MeV/nucleon} = Mass(Na)v^2/2 + Mass(n)v^2/2 = [20Mass(n) + Mass(n)] \cdot v^2/2$
The sodium thus takes 20/21 of the available kinetic energy, which is 20/21 × (25 MeV/nucleon × 20 nucleons) = 476 MeV, or 23.8 MeV/nucleon.

(d) A magnetic field can be used to separate the through-going primary beam and the desired reaction products. Discuss the principle behind this technique. (5 points) Different charges and momentum; similar to mass spectrometer.

3 Decay & Symmetry (20 Points)

The ^{20}Na nuclei produced in the experiment described above are stopped inside a detector setup where they are spin-polarized. In 100% of the decays a β with a maximum energy of 10 MeV is produced. In 20% of the decays an additional α particle is produced and in the remaining 80% an additional 1.6 MeV photon.



(a) Draw a Feynman diagram at the nucleon level to illustrate the β decay of ^{20}Na. What is the charge of the β? (2 points) From the isotope table we can see that ^{20}Na is on the neutron deficient side of the stable region. This means that if it decays a proton is turned into a neutron and hence that the beta must carry away one positive unit of EM charge; thus β^+. After β-decay the final state nucleus (^{20}Ne) is apparently in an excited state because additional particles are formed.

(b) Which interaction is responsible (dominates) for each of the decays? Motivate. (4 points) Emission of a β and corresponding neutrino is due to the weak interaction.

Emission of α particle is due to the strong interaction. Photon emission is due to EM interaction.

(c) Identify the nuclei ${}^A_Z X$ and ${}^A_Z Y$ that are produced in the decay of ^{20}Na. Explain why α emission after β decay is possible (comment on the stability of any intermediate states). (2 points) The following reactions take place:
${}^{20}_{9}\mathrm{Na} \rightarrow {}^{20}_{10}\mathrm{Ne}^* + e^+ + \nu_e$
${}^{20}_{10}\mathrm{Ne}^* \rightarrow {}^{4}_{2}\mathrm{He} + {}^{16}_{8}\mathrm{O}$
${}^{20}_{10}\mathrm{Ne}^* \rightarrow {}^{20}_{10}\mathrm{Ne} + \gamma$
Hence the final nuclei that are produced are ^{20}Ne, ^{16}O and ^{4}He (the alpha-particle). Alpha-decay is possible because ^{20}Ne is formed in an excited state. When it is in the ground state, ^{20}Ne is stable and would never decay to ^{16}O.

(d) The ^{20}Na ($t_{1/2} \simeq 0.5\,\mathrm{s}$) is produced in a short burst ($T \ll t_{1/2}$). What is the nuclear composition 2 s afterwards? (6 points) Electro-magnetic and strong decay usually proceed much faster than weak decay. You may therefore assume that the second decay step proceeds essentially instantaneously. Assume that $N_{20Na}(t = 0) = N_0$. After time t $N_{20Na}(t) = N_0 e^{-t/\tau}$, with τ the exponential lifetime, for which $\tau = t_{1/2}/\ln(2) = 0.5/\ln(2) = 0.72s$. So $N_{20Na}(t = 2s) = N_0 e^{-2/0.72} = 0.062 N_0$. The remainder is devided between ^{20}Ne and ^{4}He: ($N_{20Ne} = 0.8 \cdot (1 - 0.062) \cdot N_0 = 0.75 N_0$) and ^{16}O and ^{4}He $N_{4He} = N_{16O} = N_0(1 - N_{20Ne}) = 0.2 \cdot (1 - 0.062) \cdot N_0 = 0.19 N_0$) each.

(e) Explain how the spatial distribution of the β's can be used to measure the polarization of the ^{20}Na's. Could you instead use the spatial distribution of the 1.6 MeV γ's? Motivate. (6 points) Weak decay violates parity. This means that the β are emitted preferentially along the spin of the parent nucleus (*c.f.* the Wu experiment that proved parity violation). Observation of an asymmetry then immediately is a proof that the average spin direction is not zero, and hence that the ^{20}Na's are polarized. The photon is emitted in an electromagnetic decay, which is parity conserving. Hence the emission pattern of the photon cannot depend on the orientation of the spin and thus cannot say anything about the polarization.

4 Scattering & Capture (20 Points)

To understand atomic parity violation, the size of the atomic nucleus becomes important. Two methods are considered to find this size: I. scattering and II. muon capture.

(a) Explain (qualitatively) how the angular distribution of scattered particles is related to the shape(s) of the nucleus. (10 points) The angular distribution of the scattered particle depends on several things: the kind of interaction, the spin of the particle and the shape of the scattering particles. We ignore the spin dependence. For a given interaction and under the assumption of point-like projectile and target the angular distribution can be calculated. The ratio between the observed and the calculated distribution is due to the shape of the target (and projectile if it is not pointlike) and is commonly called the form factor. The form factor is the Fourier transform of the distribution of scattering centers.

(b) Compare α and electron scattering. Which aspects of the nuclear shape(s) you are sensitive to in each case. Which do you prefer and why? (4 points) Alpha particles interact strongly whereas electrons only interact via the EM interaction. Hence the α probes both protons and neutrons and electrons only the protons (and perhaps the magnetic moments of the neuterons). As the α also has a shape, the measured form factor is a combination the shapes of the scattering target and the projectile. In the case of electron scattering the form factor is solely determined by the target.

(c) Does the experimental sensitivity change if you were to scatter muons instead of electrons? Of would is merely be an experimental complication? (2 points) The de Broglie wavelength of muons is shorter for the same total energy so that the "resolving power" is better. However, muons decay and must be produced first in some reaction. It will seriously complicate the experiment.

(d) Negative muons may also capture on the nucleus. Draw the corresponding Feynman diagram (at the nucleon level). (2 points) Negative muons and proton in the initial state, W-boson in the intermediate state, neutrino and neutron in the final state.

(e) A μ^- may form a bound state with the nucleus, where it quickly ends up in the $1S$ ground state. Only there it will have a significant capture probability. Give the reason why the muon exhibits a larger capture rate in S-states than other states. (2 points) Muon capture is a weak interaction process, mediated by a heavy W boson. The range of this interaction is very short. The muon and the nucleus thus must come in very close contact, *i.e.* the wave function of the muon must overlap with the nucleus. Only for particles in a (relative) S-state this is the case. For all other states, the wave function has a zero at the origin.

5 Mass Formula (15 Points)

The binding energy of an α-particle is 28.3 MeV. In the framework of the liquid drop model (see Appendix) you can estimate from which mass number A onward α-decay is possible for all nuclei. Explain which steps are necessary to calculate this value. What is this value? You may neglect terms such as atomic binding and pairing energies (please explain why that makes sense). You may use $N = Z$ (is that reasonable?).

Solution. Decay via α emission is possible if the mass of the parent nucleus exceeds the mass of the daughter nucleus and the mass of the α particle:

$$M(A, N) > M(A - 4, N - 2) + M(4, 2)$$

or

$$\Delta(A,N) = M(A,N) - M(A-4,N-2) > M(4,2) = 2m_p + 2m_n + 2m_e - 28.3\,\mathrm{MeV}/c^2.$$

Atomic binding is at the eV scaler, whereas nuclear binding is at the MeV scale. It may thus be ignored. The pairing energy is relevant if the oddness or evenness of the parent and daughter nuclei differs (which is not the case) and is furthermore inversely proportional to $\sqrt{A}$ and thus increasingly small for larger A. We thus find

$$M(A,Z) = Z(m_p + m_e) + (A-Z)m_n - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(Z-A/2)^2}{A}$$

The first three terms of $M(A,Z)$ give for the lefthand side minus the proton, neutron and electron mass of the righthand side

$$\Delta = -4a_1 = -4 \cdot 15.67 = -62.68\,\mathrm{MeV}/c^2$$

For the remaining terms we have to find

$$\Delta_2 + \Delta_3 + \Delta_4 > 34\,\mathrm{MeV}/c^2$$

with

$$\begin{aligned}\Delta_2 &= a_2[A^{2/3} - (A-4)^{2/3}] \\ \Delta_3 &= a_3[\tfrac{Z^2}{A^{1/3}} - \tfrac{(Z-2)^2}{(A-4)^{1/3}}] \\ \Delta_4 &= a_4[\tfrac{(Z-N)^2}{4A} - \tfrac{(Z-N)^2}{4(A-4)}]\end{aligned}$$

For light nuclei we have $N \simeq Z$. For heavier ones in general $N > Z$, upto about a factor 1.5. The choice $N = Z$ is thus fairly reasonable as long as A is not too large. For $A = 2Z$ we find

$$\begin{aligned}\Delta_2 &= a_2[A^{2/3} - (A-4)^{2/3}] \\ \Delta_3 &= a_3\tfrac{1}{4}[A^{5/3} - (A-4)^{5/3}] \\ \Delta_4 &= 0\end{aligned}$$

From the appendix we have $a_2 = 17.23\,\mathrm{MeV}/c^2$ and $a_3 = 0.714\,\mathrm{MeV}/c^2$. Δ grows monotinically, so we find find $A = 90$ for which α decay occurs by trial and error. Alternatively, we can use the approximation $(x-d)^n \simeq x^n - x^{n-1}d + \cdots$ to arrive at $\frac{8}{3}a_2/A^{1/3} + \frac{10}{6}a_3 A^{2/3} > 34$ with a similar answer.

Constants

Speed of light	c	$2.998 \cdot 10^{8}$	m/s
Planck constant	h	$4.136 \cdot 10^{-24}$	GeV·s
	$\hbar = \frac{h}{2\pi}$	$6.582 \cdot 10^{-25}$	GeV/c
Electron charge	e	$1.602 \cdot 10^{-19}$	C
Electron mass	m_e	0.510998918(44)	MeV/c^2
Proton mass	m_p	938.272029(80)	MeV/c^2
Neutron mass	m_n	939.565360(81)	MeV/c^2
Deuteron mass	m_d	1875.61282(16)	MeV/c^2
Alpha particle mass	m_α	3727.37917(32)	MeV/c^2
Electron neutrino mass	m_{ν_e}	< 2.2	eV/c^2
Muon mass	m_μ	105.658369(9)	MeV/c^2
Tau mass	m_τ	1776.84(17)	MeV/c^2
Charged pion mass	$m_{\pi^\pm}$	139.57018(35)	MeV/c^2
Neutral pion mass	m_{π^0}	134.9766(6)	MeV/c^2
$W^\pm$-boson mass	m_W	80.403(29)	MeV/c^2
Z^0-boson mass	m_W	91.1876(21)	MeV/c^2
Avogadro's number	N_A	$6.02214179(30) \cdot 10^{23}$	mol^{-1}

Semi-Emperical Mass Formula (Bethe-Weizsäcker)

$$M(A,Z) = Nm_n + Zm_p - a_v A + a_s A^{2/3} + a_c \frac{Z^2}{A^{1/3}} + a_a \frac{(A-2Z)^2}{4A} + \frac{\delta}{A^{1/2}}$$

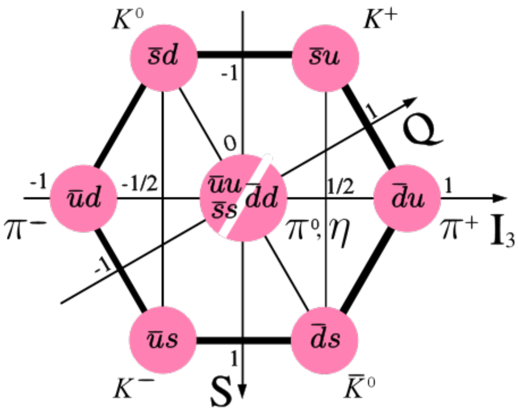
$$\begin{aligned} a_v &= 15.67 \ \mathrm{MeV}/c^2 \\ a_s &= 17.23 \ \mathrm{MeV}/c^2 \\ a_c &= 0.714 \ \mathrm{MeV}/c^2 \\ a_a &= 93.15 \ \mathrm{MeV}/c^2 \\ \delta &= 0 \ \text{odd } A \\ &= -11.2 \ \mathrm{MeV}/c^2, \quad Z \text{ and } N \text{ even} \\ &= +11.2 \ \mathrm{MeV}/c^2, \quad Z \text{ and } N \text{ odd} \end{aligned}$$

Conversion Factors

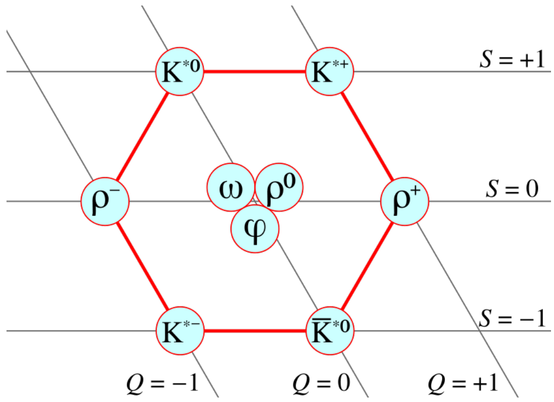
Electronvolt	eV	$1.60217653(14) \cdot 10^{-19}$	J
Tesla	T	$0.561 \cdot 1030$	MeV/c^2·C·s
kilogram	kg	$5.60958896(48) \cdot 10^{35}$	eV/c^2
barn	b	$1 \cdot 10^{-28}$	m^2

Note: For some of the questions different approaches are possible, such that you may not necessarily need all of the given constants and equations. Unless specifically stated, the final results are sufficient if given to 2 significant figures (2 leading digits).

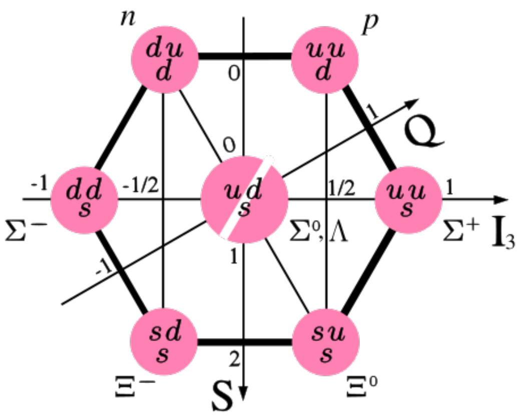
Baryon and Meson Composition



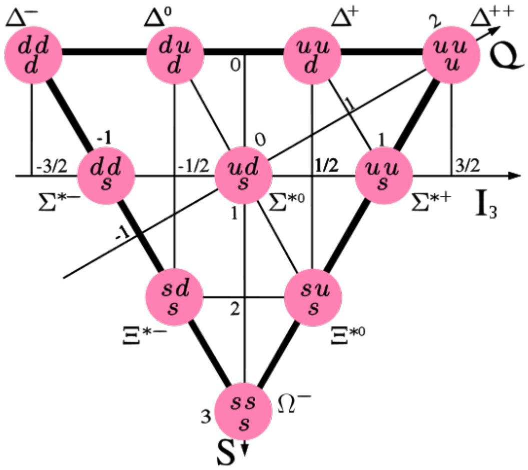
Spin-0 Mesons



Spin-1 Mesons



Spin-1/2 Baryons



Spin-3/2 Baryons

name	composition	mass [GeV/c^2]
J/ψ	$c\bar{c}$	3097
D^+	$c\bar{d}$	1869
D^0	$c\bar{u}$	1864
$\bar{D}^0$	$\bar{c}u$	1864
D^-	$\bar{c}d$	1869

Isotope Table near ^{20}Na

	1		3		5		7		9		11		13		15		N
7			10N	11N	12N	13N	14N	15N	16N	17N	18N	19N	20N	21N	22N	23N	24N
				12O	13O	14O	15O	16O	17O	18O	19O	20O	21O	22O	23O	24O	25O
9					14F	15F	16F	17F	18F	19F	20F	21F	22F	23F	24F	25F	26F
						16Ne	17Ne	18Ne	19Ne	20Ne	21Ne	22Ne	23Ne	24Ne	25Ne	26Ne	27Ne
11							18Na	19Na	20Na	21Na	22Na	23Na	24Na	25Na	26Na	27Na	28Na
							19Mg	20Mg	21Mg	22Mg	23Mg	24Mg	25Mg	26Mg	27Mg	28Mg	29Mg
13								21Al	22Al	23Al	24Al	25Al	26Al	27Al	28Al	29Al	30Al
								22Si	23Si	24Si	25Si	26Si	27Si	28Si	29Si	30Si	31Si
Z									24P	25P	26P	27P	28P	29P	30P	31P	32P